INTEGRATION

EXERCISE 7.1

How differentiation and Integration Works

$f(x): x^3$

Differentiating both sides

$f'(x): nx^{n-1} = 3x^{3-1} = 3x^2$

$f'(x): = 3x^2$

Integrating both sides

$f(x): 3\ \dfrac{x^{2+1}}{2+1} = x^3$

Formula:-

$x^n = nx^{n-1}$

Formula:–

$x^n = \dfrac{x^{n+1}}{n+1}$

$$\int x^n = \frac{x^{n+1}}{n+1} + c \ , \ n \neq -1$$

Example:

$$\int x^3 = (\text{Here } n = 3) = \frac{x^{3+1}}{3+1} = \frac{1}{4}x^4$$

$$\int x^{-3} = (\text{Here } n = -3) = \frac{x^{-3+1}}{-3+1} = \frac{-1}{2}x^{-2} = \frac{-1}{2x^2}$$

$$\int (ax+b)7 = (\text{Here } n = 7) = \frac{(ax+b)7+1}{7+1} * \frac{1}{a} = \frac{1}{8a}$$

$(ax+b)8$

Example:

$$\int x^3 = \frac{x^{3+1}}{3+1} = \frac{1}{4}x^4 = \frac{4}{4}x^3 = x^3$$

$$\int x^{-3} = \frac{x^{-3+1}}{-3+1} = \frac{1}{-2}x^{-2} = \frac{1}{-2} * \frac{1}{x^2} = \frac{-1}{2} * \frac{1}{x^2} = \frac{-1}{2x^2}$$

$$\int (ax+b)7 = \frac{(ax+b)7+1}{7+1} * \frac{1}{a} = \frac{1}{8a}(ax+b)8$$

INTEGRATION OF TRIGNOMETRIC FUNCTION

Formulae:–

$\int \sin x \, dx = -\cos x + C$

$\int \cos x \, dx = \sin x + C$

$\int \sec x \, dx = \log(\sec x + \tan x) + C$

$\int \operatorname{cosec} x \, dx = \log(\operatorname{cosec} x - \cot x) + C$

$\int \tan x \, dx = -\log(\cos x) + C = \log(\sec x) + C$

$\int \cot x \, dx = \log(\sin x) + C = -\log(\operatorname{cosec} x) + C$

Formulae:–

$\int Sec^2x\ dx = Tan\ x + C$

$\int Cosec^2x\ dx = -Cot\ x + C$

$\int Sec\ x\ Tan\ x\ dx = Sec\ x + C$

$\int Cosec\ x\ Cot\ x\ dx = -Cosec\ x + C$

Formulae:–

$\int Sec^2x\ dx = Tan\ x + C$

$\int Cosec^2x\ dx = -Cot\ x + C$

$\int Sec\ x\ Tan\ x\ dx = Sec\ x + C$

$\int Cosec\ x\ Cot\ x\ dx = -Cosec\ x + C$

Formulae:–

$$\int a^x\,dx = \frac{a^x}{\text{Log } a} + C$$

Example: $\int 3^x = \frac{3^x}{\log 3} + c$

$$\int 1\,dx = X + c$$

$$\int e^{ax}\,dx = \frac{e^{ax}}{a} + C$$

Example: $\int e^{2x} = \frac{1}{2} e^{2x}$

INTEGRATION

Exercise 7.1

Q1: $\int \sin 2x \, dx = -\frac{\cos 2x}{2} + c$

as $\int \text{Sin } x \, dx = -\text{Cos } x + C$

Q2: $\int \cos 3x \, dx = \frac{\sin 3x}{3} + c$

as $\int \text{Cos } x \, dx = \text{Sin } x + C$

Q3: $\int e^{2x} = \frac{e^{2x}}{2} + c$

as $\int e^x \, dx = e^x + C$

Note: The constant with the x will always come in the denominator in the solution.

$$Q4: \int (ax+b)^2 dx = \frac{(ax+b)^{2+1}}{2+1} * \frac{1}{a} + c = \frac{(ax+b)^3}{3a} + c$$

as $x^n = \frac{x^{n+1}}{n+1}$ and here x is (ax+b)

Also the constant with the x (which is a in the Q) will always come in the denominator in the solution.

$$Q5: \int (\sin 2x - 4e^{3x}) dx = \frac{-\cos 2x}{2} - \frac{4e^{3x}}{3} + c$$

Q6: $\int (4e^{3x} + 1)\,dx$

$= 4\int e^{3x}\,dx + \int 1\,dx$

$= 4\frac{e^{3x}}{3} + x + c = \frac{4e^{3x}}{3} + x + c$

Q7: $\int x^2 (1 - \frac{1}{x^2})\,dx$

$= \int x^2 (\frac{x^2 - 1}{x^2})\,dx$ (taking the LCM)

$= \int (x^2 - 1)\,dx = \frac{x^3}{3} - x + c$

Q8: $\int (ax^2 + bx + c)\,dx$

$= a\int x^2\,dx + b\int x\,dx + \int c\,dx = a\frac{x^3}{3} + b\frac{x^2}{2} + cx + d$

$= \frac{ax^3}{3} + \frac{bx^2}{2} + cx + C$

Q9: $\int(2x^2 + e^x)dx = \frac{2x^3}{3} + e^x + c$

Q10: $\int(\sqrt{x} - \frac{1}{\sqrt{x}})^2 dx$

$= \int x + \frac{1}{x} - 2$

$= \int(x + \frac{1}{x} - 2)dx \quad = \frac{x^2}{2} + \log x - 2x + c$

Q11: $\int \frac{x^3+5x^2-4}{x^2}dx$

$$= \int \frac{x^3}{x^2} + \frac{5x^2}{x^2} - \frac{4}{x^2}dx = \int\left(x+5-\frac{4}{x^2}\right)dx = \int\left(x+5-4x^{-2}\right)dx$$

$$= \frac{x^2}{2} + 5x - 4\frac{x^{-2+1}}{-2+1} + c = \frac{x^2}{2} + 5x - 4\frac{x^{-1}}{-1} + c$$

$$= \frac{x^2}{2} + 5x + \frac{4}{x} + c$$

$$Q12: \int \frac{x^3+3x+4}{\sqrt{x}}\,dx$$

$$= \int \frac{x^3}{\sqrt{x}} + \frac{3x}{\sqrt{x}} + \frac{4}{\sqrt{x}}\,dx$$

$$= \int \frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}}\,dx$$

$$= \int (x^{3-1/2} + 3x^{1-1/2} + 4x^{-1/2})\,dx$$

$$= x^{5/2} + 3x^{1/2} + 4x^{-1/2} + c$$

$$= \int \frac{x^{5/2+1}}{\frac{5}{2}+1} + \frac{3x^{1/2+1}}{\frac{1}{2}+1} + \frac{4x^{-1/2+1}}{-\frac{1}{2}+1} + c$$

$$= \frac{2}{7}x^{7/2} + 3*\frac{2}{3}x^{3/2} + 4*\frac{2}{1}x^{1/2} + c$$

$$= \frac{2}{7}x^{7/2} + 2x^{3/2} + 8\sqrt{x} + c$$

Q13: $\int \frac{x^3 - x^2 + x - 1}{x - 1}\, dx = \int \frac{x^2(x-1) + 1(x-1)}{x - 1}\, dx$

$= \int \frac{(x^2+1)(x-1)}{x - 1}\, dx \quad = \int (x^2 + 1) dx \quad = \frac{x^3}{3} + x + c$

Q14: $\int(1-x)\sqrt{x}\,dx = \int(x^{1/2} - x^{3/2})\,dx$

$= \frac{x^{½+1}}{½+1} - \frac{x^{3/2+1}}{3/2+1} + c = \frac{2}{3}x^{3/2} - \frac{2}{5}x^{5/2} + c$

Q15: $\int\sqrt{x}(3x^2+2x + 3)\,dx$

$=\int(3x\ 2 + 1/2 + 2\ x^{1/2+1} + 3x^{1/2})\,dx$

$= \int 3x^{5/2} + 2x^{3/2} + 3x^{1/2})\,dx$

$= 3 * \frac{2}{7}x^{7/2} + 2*\frac{2}{5}x^{5/2} + 3*\frac{2}{3}x^{3/2} + c$

$= \frac{6}{7}x^{7/2} + \frac{4}{5}x^{5/2} + 2x^{3/2} + c$

Q16: $\int(2x - 3\cos x + e^{x})dx = \frac{2x^2}{2} - 3\sin x + e^{x} + c$

$= x^2 - 3\sin x + e^{x} + c$

Q17: $\int(2x^2 - 3\sin x + 5\sqrt{x})dx = \frac{2x^3}{3} + 3\cos x + 5\,\frac{x^{1/2+1}}{1/2+1} + c$

$= \frac{2x^3}{3} + 3\cos x + 5 * \frac{2}{3} x^{1/2+1} + c$

$= \frac{2x^3}{3} + 3\cos x + \frac{10}{3} x^{3/2} + c$

Q18: $\int(\sec x(\sec x+\tan x)dx = \int(\sec^2 x+\sec x\tan x)dx$

$= \tan x + \sec x + c$

Q19: $\int \frac{\sec^2 x}{\csc^2 x}dx = \int \frac{\int \frac{1}{\cos^2 x}dx}{\int \frac{1}{\sin^2 x}dx}dx = \int \frac{\sin^2 x}{\cos^2 x}dx = \int \tan^2 x dx$

$\int(\sec^2 x - 1)dx$

$= \tan x - x + c$

Q20: $\int \frac{2-3\sin x}{\cos^2 x}dx = \int \frac{2}{\cos^2 x} - \frac{3\sin x}{\cos^2 x}dx$

$= \int 2\sec^2 x\, dx - 3\int \tan x \sec x\, dx = 2\tan x - 3\sec x + c$

Q21: The antiderivative of $(\sqrt{x} + \frac{1}{\sqrt{x}})$ equals

$\frac{x^{½+1}}{½+1} + \frac{x^{-1/2+1}}{-1/2+1} + c = \frac{2}{3}x^{3/2} + 2x^{1/2} + c$

Q22: $\frac{d}{dx} f(x) = 4x^3 - \frac{3}{x^4}$

$f(x) = \int (4x^3 - 3x^{-4})dx = x^4 + \frac{1}{x^3} + c$

Now f (2) =0 (Given) $f(2) = 2^4 + \frac{1}{2^3} + c = 0$ $c = -\frac{129}{8}$

$f(x) = x^4 + \frac{1}{x^3} - \frac{129}{8}$